Dynamic Power Allocation in MIMO Fading Systems Without Channel Distribution Information

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***Abstract*—This paper considers dynamic power allocation in MIMO fading systems with unknown channel state distributions. First, the ideal case of perfect instantaneous channel state information at the transmitter (CSIT) is treated. Using the drift-plus-penalty method, a dynamic power allocation policy is developed and shown to approach optimality, regardless of the channel state distribution and without requiring knowledge of this distribution. Next, the case of delayed and quantized channel state information is considered. Optimal utility is fundamentally different in this case, and a different online algorithm is developed that is based on convex projections. The proposed algorithm for this delayed-CSIT case is shown to have an $O(\delta)$ optimality gap, where δ is the quantization error of CSIT.**

I. Introduction

During the past decade, the multiple-input multiple-output (MIMO) technique has been recognized as one of the most important techniques for increasing the capabilities of wireless communication systems. In the wireless fading channel, where the channel changes over time, the problem of power allocation is to determine the transmit covariance of the transmitter to maximize the ergodic capacity subject to both long term and short term power constraints. It is often reasonable to assume that instantaneous channel state information (CSI) is available at the receiver through training. Most works on power allocation in MIMO fading systems also assume that statistical information about the channel state, referred to as channel distribution information (CDI), is available at the transmitter. Under the assumption of perfect instantaneous channel state information at the receiver (CSIR) and perfect channel distribution information at the transmitter (CDIT), prior work on power allocation in MIMO fading systems can be categorized into two cases:

- Perfect instantaneous channel state information at the transmitter (ideal-CSIT): In the ideal case of perfect CSIT, optimal power allocation is known to be a water-filling solution [1]. Computation of water-levels involves a one-dimensional integral equation for fading channels with i.i.d. Rayleigh entries or a multi-dimensional integral equation for general fading channels [2].
- No CSIT: If CSIT is unavailable, the optimal power allocation is in general still open. If the channel matrix has i.i.d. Rayleigh entries, then the optimal power allocation is known to be the identity transmit covariance scaled to satisfy the power constraint [1]. The optimal power allocation in MIMO fading channels with correlated Rayleigh entries is obtained in [3], [4]. The power allocation in MIMO fading channels is further considered in [5] under a more general channel correlation model.

This prior work relies on accurate CDIT and/or on restrictive channel distribution assumptions. It can be difficult to accurately estimate the CDI, especially when there are complicated correlations. Solutions that base decisions on perfect CDIT can be suboptimal due to mismatches. This paper designs algorithms that do not require prior knowledge of the channel distribution, yet perform arbitrarily close to the optimal value that can be achieved by having this knowledge. Further, the convergence time is computed and shown to be significantly smaller than the time required to accurately estimate the channel distribution.

The ideal-CSIT assumption is reasonable in time-division duplex (TDD) systems with symmetric TDD wireless channels. However, in frequency-division duplex (FDD) scenarios and other scenarios without channel symmetry, the CSI must be estimated at the receiver, quantized, and reported back to the transmitter with a time delay. This paper first considers the ideal-CSIT case and develops a solution that does not require CDIT. Next, the case of delayed and quantized CSIT is considered and a fundamentally different algorithm is developed for that case. The latter algorithm again does not use CDIT, but achieves a utility within $O(\delta)$ of the best utility that can be achieved even with perfect CDIT, where δ is the quantization error. This shows that delayed but accurate CSIT (with $\delta \approx 0$) is almost as good as having perfect CDIT.

A. Related work and our contributions

In the ideal CSIT case, the proposed dynamic power allocation policy is an adaption of the general drift-plus-penalty algorithm for stochastic network optimization [6], [7]. In this MIMO context, the current paper shows the algorithm provides strong sample path and convergence time guarantees. The dynamic of the drift-plus-penalty algorithm is similar to that of the stochastic dual subgradient algorithm, although the optimality analysis and performance bounds are different. The stochastic dual subgradient algorithm has been applied in optimization of the wireless fading channel without CDI, e.g., downlink power scheduling in single antenna cellular systems [8], power allocation in single antenna broadcast OFDM channels [9], scheduling and resource allocation in

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random access channels [10], power allocation in multi-carrier MIMO networks [11].

In the delayed and quantized CSIT case, the situation is similar to the scenario of online convex optimization [12] except that we are unable to observe true history reward functions due to channel quantization. The proposed dynamic power allocation policy can be viewed as an online algorithm with inaccurate history information. The current paper analyzes the performance loss due to CSI quantization error and provides strong sample path and convergence time guarantees of this algorithm. According to the authors' knowledge, online convex optimization with inaccurate history information has not been studied before. The analysis in this MIMO context can be extended to more general online convex optimization with inaccurate history information. Online optimization has been applied in power allocation in wireless fading channels without CDIT and with delayed and accurate CSIT, e.g., suboptimal online power allocation in single antenna single user channels [13], suboptimal online power allocation in single antenna multiple user channels [14]. A close related recent work is [15], where online power allocation in MIMO systems is considered. The online algorithm in [15] is different from our algorithm and follows a matrix exponential learning scheme requiring the computation of matrix exponentials at each slot. In contrast, our online algorithm only involves a simple projection at each slot and a closed-form solution of this projection is derived in this paper. Work [15] also considers the effect of imperfect CSIT by assuming CSIT is unbiased, i.e., expected CSIT error conditional on observed previous CSIT is zero. This assumption of imperfect CSIT is suitable to model the CSIT measurement error or feedback error but can not capture the CSI quantization error. In contrast, the current paper only requires that CSIT error is bounded.

II. Signal model and problem formulations

A. Signal model

Consider a point-to-point MIMO fading channel that operates in slotted time with normalized time slots $t \in \{0, 1, 2, \dots\}$. There are N_T antennas at the transmitter and N_R antennas at the receiver. The channel can be modeled as

$$\mathbf{y}(t) = \mathbf{H}(t)\mathbf{x}(t) + \mathbf{z}(t)$$

where $t \in \{0, 1, 2, \dots\}$ is the time index, $\mathbf{z}(t) \in \mathbb{C}^{N_R}$ is the additive noise vector, $\mathbf{x}(t) \in \mathbb{C}^{N_T}$ is the transmitted signal vector, $\mathbf{H}(t) \in \mathbb{C}^{N_R \times N_T}$ is the channel matrix, and $\mathbf{y}(t) \in \mathbb{C}^{N_R}$ is the received signal vector. Assume that noise vectors $\mathbf{z}(t)$ are independent and identically distributed (i.i.d.) over time slots t and are normalized circularly symmetric complex Gaussian random vectors with $\mathbb{E}[\mathbf{z}(t)\mathbf{z}^H(t)] = \mathbf{I}_{N_R}$, where $\mathbf{I}_{N_R}$ denotes an $N_R \times N_R$ identity matrix.[1] Assume that channel matrices $\mathbf{H}(t)$ are i.i.d. across time t and have a fixed but arbitrary probability distribution, possibly one with correlations between entries of the matrix. Assume there exists a constant $B > 0$ such that $\|\mathbf{H}\|_F \leq B$ with probability one, where $\|\cdot\|_F$ denotes the Frobenius norm of matrices.[2]

Assume that the receiver can track channel matrices $\mathbf{H}(t)$ exactly through training. In symmetric TDD scenarios, it is reasonable to assume the transmitter has perfect CSIT. In more general scenarios, the channel matrix $\mathbf{H}(t)$ is measured at the receiver at each slot t, a quantized version $\widetilde{\mathbf{H}}(t)$ is created as a function of $\mathbf{H}(t)$, and this quantized version is fed back to the transmitter with one slot of delay. We assume that the quantization error is bounded, i.e., there exists $\delta > 0$ such that $\|\widetilde{\mathbf{H}}(t) - \mathbf{H}(t)\|_F \leq \delta$ for all t. Due to the one slot delay, at slot t the transmitter only knows $\widetilde{\mathbf{H}}(t-1)$. Since channels are i.i.d. over slots, this delayed information is independent of the current (and unknown) $\mathbf{H}(t)$. Remarkably, it turns out that the outdated information is still useful.

B. Optimal power allocation with perfect CDIT

If the channel matrix is fixed at $\mathbf{H}$ and the transmit covariance is fixed at $\mathbf{Q}$, the MIMO capacity is given by [1]:

$$\log\det(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H)$$

where superscript H denotes Hermitian transpose and $\det(\cdot)$ denotes the determinant operator of matrices. If $\mathbf{H}$ is random then the average capacity, formally called the ergodic capacity [16], is given by $\mathbb{E}_{\mathbf{H}}\big[\log\det(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H)\big]$. We consider two types of power constraints at the transmitter: An average power constraint $\mathbb{E}_{\mathbf{H}}[\text{tr}(\mathbf{Q})] \leq \bar{P}$ and an instantaneous power constraint $\text{tr}(\mathbf{Q}) \leq P$, where $\text{tr}(\cdot)$ denotes the trace operator of matrices. The ideal-CSIT problem is to choose $\mathbf{Q}$ as a (possibly random) function of the observed $\mathbf{H}$ to maximize the ergodic capacity subject to power constraints:

$$\begin{aligned} \max_{\mathbf{Q}(\mathbf{H})} \quad & \mathbb{E}_{\mathbf{H}}\big[\log\det(\mathbf{I} + \mathbf{H}\mathbf{Q}(\mathbf{H})\mathbf{H}^H)\big] && (1)\\ \text{s.t.} \quad & \mathbb{E}_{\mathbf{H}}[\text{tr}(\mathbf{Q}(\mathbf{H}))] \leq \bar{P}, && (2)\\ & \mathbf{Q}(\mathbf{H}) \in \mathcal{Q}, \forall \mathbf{H}, && (3) \end{aligned}$$

where $\mathcal{Q}$ is a set that enforces the instantaneous power constraint:

$$\mathcal{Q} = \big\{\mathbf{Q} \in \mathbb{S}_+^{N_T} : \text{tr}(\mathbf{Q}) \leq P\big\} \quad (4)$$

where $\mathbb{S}_+^{N_T}$ denotes the $N_T \times N_T$ positive semidefinite matrix space. To avoid trivialities it is assumed that $P \geq \bar{P}$. In (1)-(3) we use notation $\mathbf{Q}(\mathbf{H})$ to emphasize that $\mathbf{Q}$ can depend on $\mathbf{H}$, i.e., adaptive to channel realizations.

If the transmitter has no CSIT, the optimal power allocation problem is different, given as follows.

$$\begin{aligned} \max_{\mathbf{Q}} \quad & \mathbb{E}_{\mathbf{H}}\big[\log\det(\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^H)\big] && (5)\\ \text{s.t.} \quad & \mathbb{E}_{\mathbf{H}}[\text{tr}(\mathbf{Q})] \leq \bar{P}, && (6)\\ & \mathbf{Q} \in \mathcal{Q}, && (7) \end{aligned}$$

where set $\mathcal{Q}$ is defined in (4). Again assume $P \geq \bar{P}$. Since the instantaneous CSIT is unavailable, the transmit covariance

[1] If the size of the identity matrix is clear, we often simply write $\mathbf{I}$.

[2] A bounded Frobenius norm always holds in the physical world because the channel attenuates the signal. Particular models such as Rayleigh and Rician fading violate this assumption in order to have simpler distribution functions.

cannot adapt to $\mathbf{H}$. By the convexity of this problem and Jensen's inequality, a randomized $\mathbf{Q}$ can be shown to be useless. It suffices to consider a constant $\mathbf{Q}$. Since $P \geq \bar{P}$, this implies the problem is equivalent to a problem that removes the constraint (6) and that changes the constraint (7) to:

$$\mathbf{Q} \in \widetilde{\mathcal{Q}} = \{\mathbf{Q} \in \mathbb{S}_{+}^{N_T} : \text{tr}(\mathbf{Q}) \leq \bar{P}\}$$

The problems (1)-(3) and (5)-(7) are fundamentally different and have different optimal objective function values. Optimality for these problems is defined by the channel distribution information (CDI). In this paper, the problems are solved via dynamic algorithms that do not require CDI. The algorithms are different for the two cases, and use different techniques.

C. Linear algebra and matrix derivatives

Recall that if $\mathbf{A} \in \mathbb{C}^{m\times n}$ and $\mathbf{B} \in \mathbb{C}^{n\times m}$ then $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$. This subsection presents additional useful facts about Frobenius norms and complex matrices. Proofs are given in [17] for completeness.

Fact 1. *For any $\mathbf{A}, \mathbf{B} \in \mathbb{C}^{m\times n}$ and $\mathbf{C} \in \mathbb{C}^{n\times k}$ we have:*

1) $\|\mathbf{A}\|_F = \|\mathbf{A}^H\|_F = \|\mathbf{A}^T\| = \|-\mathbf{A}\|_F$.
2) $\|\mathbf{A}+\mathbf{B}\|_F \leq \|\mathbf{A}\|_F + \|\mathbf{B}\|_F$.
3) $\|\mathbf{AC}\|_F \leq \|\mathbf{A}\|_F\|\mathbf{C}\|_F$.
4) $|tr(\mathbf{A}^H\mathbf{B})| \leq \|\mathbf{A}\|_F\|\mathbf{B}\|_F$.

Fact 2. *For any $\mathbf{A} \in \mathbb{S}_{+}^{n}$ we have $\|\mathbf{A}\|_F \leq tr(\mathbf{A})$.*

Fact 3 ([18])**.** *The function $f : \mathbb{S}_{+}^{n} \to \mathbb{R}$ defined by $f(\mathbf{Q}) = \log\det(\mathbf{I} + \mathbf{HQH}^H)$ is concave and its gradient is given by $\nabla_{\mathbf{Q}} f(\mathbf{Q}) = \mathbf{H}^H(\mathbf{I}+\mathbf{HQH}^H)^{-1}\mathbf{H}, \forall \mathbf{Q} \in \mathbb{S}_{+}^{n}$.*

The next fact is the complex matrix version of the first order condition for concave functions of real number variables, i.e., $f(y) \leq f(x) + f'(x)(y-x), \forall x, y \in \text{dom} f$ if f is concave.

Fact 4. *Let function $f(\mathbf{Q}) : \mathbb{S}_{+}^{n} \to \mathbb{R}$ be a concave function and have gradient $\nabla_{\mathbf{Q}} f(\mathbf{Q}) \in \mathbb{S}^n$ at point $\mathbf{Q}$. Then, $f(\widehat{\mathbf{Q}}) \leq f(\mathbf{Q}) + tr\big([\nabla_{\mathbf{Q}} f(\mathbf{Q})]^H(\widehat{\mathbf{Q}} - \mathbf{Q})\big), \forall \widehat{\mathbf{Q}} \in \mathbb{S}_{+}^{n}$.*

III. Ideal CSIT case

Consider the case of perfect instantaneous CSIT, called the ideal-CSIT case. The problem to solve is (1)-(3). At the beginning of each slot $t \in \{0, 1, 2, \ldots\}$ the channel $\mathbf{H}(t)$ is known and a covariance matrix $\mathbf{Q}(t)$ can be chosen based on this information. This is done without using CDI via the *drift-plus-penalty* technique of [7]. For each slot $t \in \{0, 1, 2, \ldots\}$ define the *reward* $R(t)$:

$$R(t) = \log\det(\mathbf{I} + \mathbf{H}(t)\mathbf{Q}(t)\mathbf{H}(t)^H) \quad (8)$$

The average power constraint (2) is enforced via a *virtual queue* $Z(t)$ with $Z(0) = 0$ and with update:

$$Z(t+1) = \max[0, Z(t) + \text{tr}(\mathbf{Q}(t)) - \bar{P}]$$

In the drift-plus-penalty algorithm, every slot t a matrix $\mathbf{Q}(t) \in \mathcal{Q}$ is selected to maximize $VR(t) - Z(t)\text{tr}(\mathbf{Q}(t))$, where V is a positive weight. This results in Algorithm 1 below.

Algorithm 1 Dynamic power allocation with ideal CSIT

Let $V > 0$ be a constant parameter and $Z(0) = 0$. At each time $t \in \{0, 1, 2, \ldots\}$, observe $\mathbf{H}(t)$ and $Z(t)$. Then do the following:

- Choose transmit covariance $\mathbf{Q}(t) \in \mathcal{Q}$ to maximize:
$$V \log\det(\mathbf{I} + \mathbf{H}(t)\mathbf{Q}(t)\mathbf{H}(t)^H) - Z(t)\text{tr}(\mathbf{Q}(t))$$
- Update $Z(t+1) = \max[0, Z(t) + \text{tr}(\mathbf{Q}(t)) - \bar{P}]$.

Define R^{opt} as the optimal average utility in (1). The value R^{opt} depends on the (unknown) distribution for $\mathbf{H}(t)$. Fix $\epsilon > 0$ and define $V = (P + \bar{P})^2/(2\epsilon)$. A theorem in [7] ensures that, regardless of the distribution of $\mathbf{H}(t)$:

$$\frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}[R(\tau)] \geq R^{opt} - \epsilon, \quad \forall t > 0 \quad (9)$$

$$\lim_{t\to\infty} \frac{1}{t}\sum_{\tau=0}^{t-1} \mathbb{E}[\text{tr}(\mathbf{Q}(\tau))] \leq \bar{P} \quad (10)$$

This holds for arbitrarily small values of $\epsilon > 0$, and so the algorithm comes arbitrarily close to optimality. Notice that Algorithm 1 does not use channel distribution information (i.e., no CDI). The next subsections show how to solve the covariance selection problem for choosing $\mathbf{Q}(t)$ in Algorithm 1, and shows that the special structure of this MIMO problem produces a sample path guarantee that is significantly stronger than (10) and that demonstrates convergence time that is typically much faster than the time that would be required to accurately estimate the CDI information.

A. Transmit covariance updates in Algorithm 1

This subsection shows the $\mathbf{Q}(t)$ selection in Algorithm 1 can be easily solved and has an (almost) closed-form solution. The convex program involved in the transmit covariance update of Algorithm 1 is in the form

$$\max_{\mathbf{Q}} \quad \log\det(\mathbf{I} + \mathbf{HQH}^H) - \frac{Z}{V}\text{tr}(\mathbf{Q}) \quad (11)$$

$$\text{s.t.} \quad \text{tr}(\mathbf{Q}) \leq P \quad (12)$$

$$\mathbf{Q} \in \mathbb{S}_{+}^{N_T} \quad (13)$$

This convex program is similar to the conventional problem of transmit covariance design with a deterministic channel $\mathbf{H}$, except that objective (11) has an additional penalty term $-(Z/V)\text{tr}(\mathbf{Q})$. It is well known that, without this penalty term, the solution is to diagonalize the channel matrix and allocate power over eigen-modes according to a water-filling technique [1]. The next theorem shows that the optimal solution to problem (11)-(13) has a similar structure.

Theorem 1. *Consider the SVD $\mathbf{H}^H\mathbf{H} = \mathbf{U}^H\mathbf{\Sigma}\mathbf{U}$, where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Sigma}$ is a diagonal matrix with non-negative entries $\sigma_1, \ldots, \sigma_{N_T}$. Then the optimal solution to* (11)-(13) *is* *given by $\mathbf{Q}^* = \mathbf{U}^H\mathbf{\Theta}^*\mathbf{U}$, where $\mathbf{\Theta}^*$ is a diagonal matrix with entries $\theta_1^*, \ldots, \theta_{N_T}^*$ given by:*

$$\theta_i^* = \max\big[0, \frac{1}{\mu^* + Z/V} - \frac{1}{\sigma_i}\big], \quad \forall i \in \{1, \ldots, N_T\},$$

where μ^* is chosen such that $\sum_{i=1}^{N_T}\theta_i^* \leq P$, $\mu^* \geq 0$ and $\mu^*\big[\sum_{i=1}^{N_T}\theta_i^* - P\big] = 0$. The exact μ^* can be determined with complexity $O(N_T \log N_T)$, described in Algorithm 2.

Proof: See Appendix A. ∎

Algorithm 2 Algorithm to solve problem (11)-(13)

1) Check if $\sum_{i=1}^{N_T}\max\{0, \frac{1}{Z/V} - \frac{1}{\sigma_i}\} \leq P$ holds. If yes, let $\mu^* = 0$ and $\theta_i^* = \max\{0, \frac{1}{Z/V} - \frac{1}{\sigma_i}\}, \forall i \in \{1, 2, \ldots, N_T\}$ and terminate the algorithm; else, continue to the next step.
2) Sort all $\sigma_i, \in \{1, 2, \ldots, N_T\}$ in a decreasing order π such that $\sigma_{\pi(1)} \geq \sigma_{\pi(2)} \geq \cdots \geq \sigma_{\pi(N_T)}$. Define $S_0 = 0$.
3) For $i = 1$ to N_T
   - Let $S_i = S_{i-1} + \frac{1}{\sigma_{\pi(i)}}$. Let $\mu^* = \frac{i}{S_i + P} - (Z/V)$.
   - If $\mu^* \geq 0$, $\frac{1}{\mu^* + Z/V} - \frac{1}{\sigma_{\pi(i)}} > 0$ and $\frac{1}{\mu^* + Z/V} - \frac{1}{\sigma_{\pi(i+1)}} \leq 0$, then terminate the loop; else, continue to the next iteration in the loop.
4) Let $\theta_i^* = \max\big[0, \frac{1}{\mu^* + Z/V} - \frac{1}{\sigma_i}\big], \forall i \in \{1, 2, \ldots, N_T\}$ and terminate the algorithm.

The complexity of Algorithm 2 is dominated by the sorting of all σ_i in step (2). Recall that the water-filling solution of power allocation in multiple parallel channels can also be found by an exact algorithm (see Section 6 in [19]), which is similar to Algorithm 2. The main difference is that Algorithm 2 has a first step to verify if $\mu^* = 0$. This is because unlike the power allocation in multiple parallel channels, where the optimal solution always uses full power, the optimal solution to problem (11)-(13) may not use full power for large Z due to the penalty term $-(Z/V)\text{tr}(\mathbf{Q})$ in objective (11).

B. Deterministic bounds

Recall that $\|\mathbf{H}(t)\|_F^2 \leq B^2$ for all t, for some constant B.

Lemma 1. *In Algorithm 1, if $Z(t) \geq VB^2$, then $\mathbf{Q}(t) = \mathbf{0}$.*

Proof: Suppose the SVD of $\mathbf{H}^H(t)\mathbf{H}(t)$ is given by $\mathbf{H}^H(t)\mathbf{H}(t) = \mathbf{U}^H\mathbf{\Sigma}\mathbf{U}$, where diagonal matrix $\mathbf{\Sigma}$ has non-negative diagonal entries $\sigma_1, \ldots, \sigma_{N_T}$. Note that $\sigma_i \overset{(a)}{\leq} \text{tr}(\mathbf{H}^H(t)\mathbf{H}(t)) \overset{(b)}{\leq} \|\mathbf{H}(t)\|_F^2 \leq B^2$ where (a) follows from $\text{tr}(\mathbf{H}^H(t)\mathbf{H}(t)) = \sum_{i=1}^{N_T}\sigma_i$; and (b) follows from Fact 1. By Theorem 1, $\mathbf{Q}(t) = \mathbf{U}^H\mathbf{\Theta}^*\mathbf{U}$, where $\mathbf{\Theta}^*$ is a diagonal matrix with entries $\theta_1^*, \ldots, \theta_{N_T}^*$ given by $\theta_i^* = \max\big[0, \frac{1}{\mu^* + Z(t)/V} - \frac{1}{\sigma_i}\big], \forall i \in \{1, 2, \ldots, N_T\}$, where $\mu^* \geq 0$.

Since $\sigma_i \leq B^2, \forall i \in \{1, 2, \ldots, N_T\}$, we know that if $Z(t) \geq VB^2$, then $\frac{1}{\mu + Z(t)/V} - \frac{1}{\sigma_i} \leq 0$ for all $\mu \geq 0$ and hence $\theta_i^* = 0, \forall i \in \{1, 2, \ldots, N_T\}$. ∎

Lemma 2. *Let $Z(t)$ be yielded by Algorithm 1. For all slots $t \in \{0, 1, 2, \ldots\}$, we have $Z(t) \leq VB^2 + (P - \bar{P})$.*

Proof: By Lemma 1, $Z(t)$ can not increase if $Z(t) \geq VB^2$. If $Z(t) \leq VB^2$, then $Z(t+1)$ is at most $VB^2 + (P - \bar{P})$ by the update equation of $Z(t+1)$ and the instantaneous power constraint. ∎

C. Performance of Algorithm 1 (ideal-CSIT)

Theorem 2. *Fix $\epsilon > 0$ and define $V = (P + \bar{P})^2/(2\epsilon)$. Under Algorithm 1 we have for all $t > 0$:*

$$\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbb{E}[R(\tau)] \geq R^{opt} - \epsilon$$
$$\frac{1}{t}\sum_{\tau=0}^{t-1} tr(\mathbf{Q}(\tau)) \leq \bar{P} + \frac{B^2(P + \bar{P})^2 + 2\epsilon(P - \bar{P})}{2\epsilon t}$$

In particular, the sample path time average power is within ϵ of its required constraint $\bar{P}$ whenever $t \geq \Omega(1/\epsilon^2)$.

Proof: The first inequality is the same as (9). It remains to prove the second inequality. For all slots τ the Algorithm 1 update for $Z(\tau)$ satisfies:

$$\begin{aligned} Z(\tau + 1) &= \max[0, Z(\tau) + \text{tr}(\mathbf{Q}(\tau)) - \bar{P}] \\ &\geq Z(\tau) + \text{tr}(\mathbf{Q}(\tau)) - \bar{P} \end{aligned}$$

Rearranging terms gives: $\text{tr}(\mathbf{Q}(\tau)) - \bar{P} \leq Z(\tau + 1) - Z(\tau)$. Fix $t > 0$. Summing over $\tau \in \{0, \ldots, t-1\}$ and dividing by t gives:

$$\begin{aligned} \frac{1}{t}\sum_{\tau=0}^{t-1}\text{tr}(\mathbf{Q}(\tau)) - \bar{P} &\leq \frac{Z(t) - Z(0)}{t} \\ &\leq \frac{1}{t}(VB^2 + (P - \bar{P})) \end{aligned}$$

where the last inequality holds because $Z(0) = 0$ and $Z(t) \leq VB^2 + (P - \bar{P})$ by Lemma 2. ∎

Theorem 2 provides a sample path guarantee on average power, which is much stronger than the gurantee in (10). It also shows that convergence time to reach an ϵ-approximate solution is $O(1/\epsilon^2)$. Typically, this is dramatically more efficient than the convergence time required to obtain even a coarse estimate of the joint distribution for the entries of $\mathbf{H}(t)$. Indeed, if each channel entry h_{ij} were quantized into $1/\delta$ distinct levels, there would be $(1/\delta)^{N_T N_R}$ different possible (quantized) matrix realizations. Waiting for $(1/\delta)^{N_T N_R}$ slots would at best allow each realization to appear once, which is still not enough for accurate estimation of the probabilities associated with each realization. Fortunately, Theorem 2 shows that such estimation is not needed.

IV. Delayed and quantized CSIT case

Consider the case of delayed and quantized CSIT. At the beginning of each slot $t \in \{0, 1, 2, \ldots\}$, channel $\mathbf{H}(t)$ is unknown and only quantized channels of previous slots $\widetilde{\mathbf{H}}(\tau), \tau \in \{0, 1, \ldots, t-1\}$ are known.

This is similar to the scenario of online optimization where the decision maker selects $x(t) \in \mathcal{X}$ at each slot t to maximize an unknown reward function $f_t(x)$ based on the information of previous reward functions $f_\tau(x(\tau)), \tau \in \{0, 1, \ldots, t-1\}$. The goal is to minimize average regret $\frac{1}{t}\max_{x\in\mathcal{X}}\big[\sum_{\tau=0}^{t-1} f_\tau(x)\big] - \frac{1}{t}\sum_{\tau=0}^{t-1} f_\tau(x(\tau))$. The best known average regret of online optimization with Lipschitz continuous and convex reward functions is $O(\frac{1}{\sqrt{t}})$ in [12].

This is different from conventional online optimization because at each slot t, the rewards of previous slots, i.e., $R(\tau) = \log\det(\mathbf{I} + \mathbf{H}(\tau)\mathbf{Q}(\tau)\mathbf{H}^H(\tau)), \tau \in \{0, 1, \ldots, t-1\}$, are still unknown due to the fact that the reported channels $\widetilde{\mathbf{H}}(\tau)$ are the quantized versions. Nevertheless, an online algorithm without using CDIT is developed in Algorithm 3.

Algorithm 3 Dynamic Power Allocation with Delayed and Quantized CSIT

Let $\gamma > 0$ be a constant parameter and $\mathbf{Q}(0) \in \mathcal{Q}$ be arbitrary. At each time $t \in \{1, 2, \ldots\}$, observe $\widetilde{\mathbf{H}}(t-1)$ and do the following:

- Let $\widetilde{\mathbf{D}}(t-1) = \widetilde{\mathbf{H}}^H(t-1)(\mathbf{I}_{N_R} + \widetilde{\mathbf{H}}(t-1)\mathbf{Q}(t-1)\widetilde{\mathbf{H}}^H(t-1))^{-1}\widetilde{\mathbf{H}}(t-1)$. Choose transmit covariance

$$\mathbf{Q}(t) = \mathcal{P}_{\widetilde{\mathcal{Q}}}\big[\mathbf{Q}(t-1) + \gamma\widetilde{\mathbf{D}}(t-1)\big],$$

  where $\mathcal{P}_{\widetilde{\mathcal{Q}}}[\cdot]$ is the projection onto convex set $\widetilde{\mathcal{Q}} = \{\mathbf{Q} \in \mathbb{S}_+^{N_T} : \text{tr}(\mathbf{Q}) \leq \bar{P}\}$.

Define $\mathbf{Q}^* \in \widetilde{\mathcal{Q}}$ as an optimal solution to problem (5)-(7), which depends on the (unknown) distribution for $\mathbf{H}(t)$. Define

$$R^{opt}(t) = \log\det(\mathbf{I} + \mathbf{H}(t)\mathbf{Q}^*\mathbf{H}^H(t))$$

as the utility at slot t attained by $\mathbf{Q}^*$.

If the channel is not quantized, i.e., $\widetilde{\mathbf{H}}(t-1) = \mathbf{H}(t-1), \forall t \in \{1, 2, \ldots\}$, then $\widetilde{\mathbf{D}}(t-1)$ is the gradient of $R(t-1)$ at point $Q(t-1)$. Fix $\epsilon > 0$ and take $\gamma = \epsilon$. The results in [12] ensure that, regardless of the distribution of $\mathbf{H}(t)$:

$$\frac{1}{t}\sum_{\tau=0}^{t-1} R(\tau) \geq \frac{1}{t}\sum_{\tau=0}^{t-1} R^{opt}(\tau) - \frac{\bar{P}}{\epsilon t} - \frac{N_R B^4}{2}\epsilon, \forall t > 0 \quad (14)$$

$$\frac{1}{t}\sum_{\tau=0}^{t-1} \text{tr}(\mathbf{Q}(\tau)) \leq \bar{P}, \forall t > 0 \quad (15)$$

The next subsections analyze the performance of Algorithm 3 with quantized channels and shows that the performance degrades linearly with respect to the quantization error δ. If $\delta = 0$, then (14) and (15) are recovered.

A. Transmit Covariance Updates in Algorithm 3

This subsection shows the $\mathbf{Q}(t)$ selection in Algorithm 3 can be easily solved and has an (almost) closed-form solution.

The projection operator involved in Algorithm 3 by definition is

$$\min \quad \frac{1}{2}\|\mathbf{Q} - \mathbf{X}\|_F^2 \quad (16)$$

$$\text{s.t.} \quad \text{tr}(\mathbf{Q}) \leq \bar{P} \quad (17)$$

$$\mathbf{Q} \in \mathbb{S}_+^{N_T} \quad (18)$$

where $\mathbf{X} = \mathbf{Q}(t-1) + \gamma\widetilde{\mathbf{D}}(t-1)$ is an Hermitian matrix at each time t.

Without constraint $\text{tr}(\mathbf{Q}) \leq \bar{P}$, the projection of Hermitian matrix $\mathbf{X}$ onto the positive semidefinite cone $\mathbb{S}_+^n$ is simply taking the eigenvalue expansion of $\mathbf{X}$ and dropping terms associated with negative eigenvalues (see Section 8.1.1. in [20]). Work [21] considered the projection onto the intersection of the positive semidefinite cone $\mathbb{S}_+^n$ and an affine subspace given by $\{\mathbf{Q} : \text{tr}(\mathbf{A}_i\mathbf{Q}) = b_i, i \in \{1, 2, \ldots, p\}, \text{tr}(\mathbf{B}_j\mathbf{Q}) \leq d_j, j \in \{1, 2, \ldots, m\}\}$ and developed the dual-based iterative numerical algorithm to calculate the projection. Problem (16)-(18) is a special case, where the affine subspace is given by $\text{tr}(\mathbf{Q}) \leq \bar{P}$, of the projection considered in [21]. Instead of solving problem (16)-(18) using numerical algorithms, this subsection shows that problem (16)-(18) has an (almost) closed-form solution.

Theorem 3. *Consider SVD $\mathbf{X} = \mathbf{U}^H\mathbf{\Sigma}\mathbf{U}$, where $\mathbf{U}$ is a unitary matrix and $\mathbf{\Sigma}$ is a diagonal matrix with entries $\sigma_1, \ldots, \sigma_{N_T}$. Then the optimal solution to problem (16)-(18) is given by $\mathbf{Q}^* = \mathbf{U}^H\mathbf{\Theta}^*\mathbf{U}$, where $\mathbf{\Theta}^*$ is a diagonal matrix with entries $\theta_1^*, \ldots, \theta_{N_T}^*$ given by,*

$$\theta_i^* = \max[0, \sigma_i - \mu^*], \forall i \in \{1, 2, \ldots, N_T\},$$

where μ^* is chosen such that $\sum_{i=1}^{N_T}\theta_i^* \leq \bar{P}$, $\mu^* \geq 0$ and $\mu^*\big[\sum_{i=1}^{N_T}\theta_i^* - \bar{P}\big] = 0$. The exact μ^* can be determined with complexity $O(N_T\log N_T)$, described in Algorithm 4.

Proof: The proof is sketched as follows. First, problem (16)-(18) is reduced to a simpler convex program with a real vector variable by characterizing the structure of its optimal solution. Then, an (almost) closed-form solution to the simpler convex program is obtained by studying its KKT conditions. See Appendix B for details. ■

Algorithm 4 Algorithm to solve problem (16)-(18)

1) Check if $\sum_{i=1}^{N_T}\max[0, \sigma_i] \leq \bar{P}$ holds. If yes, let $\mu^* = 0$ and $\theta_i^* = \max[0, \sigma_i], \forall i \in \{1, 2, \ldots, N_T\}$ and terminate the algorithm; else, continue to the next step.
2) Sort all $\sigma_i, \in \{1, 2, \ldots, N_T\}$ in a decreasing order π such that $\sigma_{\pi(1)} \geq \sigma_{\pi(2)} \geq \cdots \geq \sigma_{\pi(N_T)}$. Define $S_0 = 0$.
3) For $i = 1$ to N_T
   - Let $S_i = S_{i-1} + \sigma_i$. Let $\mu^* = \frac{S_i - \bar{P}}{i}$.
   - If $\mu^* \geq 0$, $\sigma_{\pi(i)} - \mu^* > 0$ and $\sigma_{\pi(i+1)} - \mu^* \leq 0$, then terminate the loop; else, continue to the next iteration in the loop.
4) Let $\theta_i^* = \max[0, \sigma_i - \mu^*], \forall i \in \{1, 2, \ldots, N_T\}$ and terminate the algorithm.

B. Property of $\widetilde{\mathbf{D}}(t-1)$

Define $\mathbf{D}(t-1) = \mathbf{H}^H(t-1)(\mathbf{I}_{N_R} + \mathbf{H}(t-1)\mathbf{Q}(t-1)\mathbf{H}^H(t-1))^{-1}\mathbf{H}(t-1)$, which is the gradient of $R(t-1)$ at point $\mathbf{Q}(t-1)$ and is unknown to the transmitter due to the unavailability of $\mathbf{H}(t-1)$. The next lemma relates $\widetilde{\mathbf{D}}(t-1)$ and $\mathbf{D}(t-1)$.

Lemma 3. *For all slots $t \in \{1, 2, \ldots\}$, we have*

1) $\|\mathbf{D}(t-1)\|_F \leq \sqrt{N_R}B^2$.
2) $\|\mathbf{D}(t-1) - \widetilde{\mathbf{D}}(t-1)\|_F \leq \psi(\delta)$, *where* $\psi(\delta) = \big(\sqrt{N_R}B + \sqrt{N_R}(B+\delta) + (B+\delta)^2 N_R \bar{P}(2B+\delta)\big)\delta$ *satisfying* $\psi(\delta) \to 0$ *as* $\delta \to 0$, *i.e.,* $\psi(\delta) \in O(\delta)$.
3) $\|\widetilde{\mathbf{D}}(t-1)\|_F \leq \psi(\delta) + \sqrt{N_R}B^2$

Proof: See full version [17] for details. ■

C. Performance of Algorithm 3

Theorem 4. *Fix $\epsilon > 0$ and define $\gamma = \epsilon$. Under Algorithm 3, we have for all $t > 0$:*

$$\frac{1}{t}\sum_{\tau=0}^{t-1} R(\tau) \geq \frac{1}{t}\sum_{\tau=0}^{t-1} R^{opt}(\tau) - \frac{\bar{P}}{\epsilon t} - \frac{(\psi(\delta) + \sqrt{N_R}B^2)^2}{2}\epsilon - 2\psi(\delta)\bar{P}$$

$$\frac{1}{t}\sum_{\tau=0}^{t-1} tr(\mathbf{Q}(\tau)) \leq \bar{P}$$

where $\psi(\delta)$ is the constant defined in Lemma 3. In particular, the sample path time average utility is within $\epsilon + 2\psi(\delta)\bar{P}$ of the optimal time average utility for problem (5)-(7) *whenever $t \geq \Omega(1/\epsilon^2)$.*

Proof: The second inequality trivially follows from the fact that $\mathbf{Q}(t) \in \widetilde{\mathcal{Q}}, \forall t \in \{0, 1, \ldots\}$. It remains to prove the first inequality. This proof extends the regret analysis of conventional online convex optimization [12] by considering inexact gradient $\widetilde{\mathbf{D}}(t-1)$.

For all slots $\tau \in \{1, 2, \ldots\}$, the transmit covariance update in Algorithm 3 satisfies:

$$\begin{aligned}
&\|\mathbf{Q}(\tau) - \mathbf{Q}^*\|_F^2 \\
=&\|\mathcal{P}_{\widetilde{\mathcal{Q}}}\big[\mathbf{Q}(\tau-1) + \gamma\widetilde{\mathbf{D}}(\tau-1)\big] - \mathbf{Q}^*\|_F^2 \\
\overset{(a)}{\leq}&\|\mathbf{Q}(\tau-1) + \gamma\widetilde{\mathbf{D}}(\tau-1) - \mathbf{Q}^*\|_F^2 \\
=&\|\mathbf{Q}(\tau-1) - \mathbf{Q}^*\|_F^2 + 2\gamma \text{tr}\big(\widetilde{\mathbf{D}}^H(\tau-1)(\mathbf{Q}(\tau-1) - \mathbf{Q}^*)\big) \\
&+ \gamma^2\|\widetilde{\mathbf{D}}(\tau-1)\|_F^2 \\
=&\|\mathbf{Q}(\tau-1) - \mathbf{Q}^*\|_F^2 + 2\gamma \text{tr}\big(\mathbf{D}^H(\tau-1)(\mathbf{Q}(\tau-1) - \mathbf{Q}^*)\big) \\
&+ 2\gamma \text{tr}\big((\widetilde{\mathbf{D}}(t-1) - \mathbf{D}(\tau-1))^H(\mathbf{Q}(\tau-1) - \mathbf{Q}^*)\big) \\
&+ \gamma^2\|\widetilde{\mathbf{D}}(\tau-1)\|_F^2,
\end{aligned}$$

where (a) follows from the non-expansive property of projections onto convex sets. Define $\Delta(t) = \|\mathbf{Q}(t+1) - \mathbf{Q}^*\|_F^2 - \|\mathbf{Q}(t) - \mathbf{Q}^*\|_F^2$. Rearranging terms in the last equation and dividing by factor 2γ implies

$$\begin{aligned}
&\text{tr}\big(\mathbf{D}^H(\tau-1)(\mathbf{Q}(\tau-1) - \mathbf{Q}^*)\big) \\
\geq&\frac{1}{2\gamma}\Delta(\tau-1) - \frac{\gamma}{2}\|\widetilde{\mathbf{D}}(\tau-1)\|_F^2 \\
&- \text{tr}\big((\widetilde{\mathbf{D}}(\tau-1) - \mathbf{D}(\tau-1))^H(\mathbf{Q}(\tau-1) - \mathbf{Q}^*)\big)
\end{aligned} \tag{19}$$

Define $f(\mathbf{Q}) = \log\det(\mathbf{I} + \mathbf{H}(\tau-1)\mathbf{Q}\mathbf{H}^H(\tau-1))$. By Fact 3, $f(\cdot)$ is concave over $\widetilde{\mathcal{Q}}$. Note that $\mathbf{D}(t-1) = \nabla_{\mathbf{Q}} f(\mathbf{Q}(t-1))$ by Fact 3 and $\mathbf{Q}^* \in \widetilde{\mathcal{Q}}$. By Fact 4, we have

$$f(\mathbf{Q}(\tau-1)) - f(\mathbf{Q}^*) \geq \text{tr}(\mathbf{D}^H(\tau-1)(\mathbf{Q}(\tau-1) - \mathbf{Q}^*)). \tag{20}$$

Note that $f(\mathbf{Q}(\tau-1)) = R(\tau-1)$ and $f(\mathbf{Q}^*) = R^{opt}(\tau-1)$. Combining (19) and (20) yields

$$\begin{aligned}
&R(\tau-1) - R^{opt}(\tau-1) \\
\geq&\frac{1}{2\gamma}\Delta(\tau-1) - \frac{\gamma}{2}\|\widetilde{\mathbf{D}}(\tau-1)\|_F^2 \\
&- \text{tr}\big((\widetilde{\mathbf{D}}(\tau-1) - \mathbf{D}(\tau-1))^H(\mathbf{Q}(\tau-1) - \mathbf{Q}^*)\big) \\
\overset{(a)}{\geq}&\frac{1}{2\gamma}\Delta(\tau-1) - \frac{\gamma}{2}\|\widetilde{\mathbf{D}}(\tau-1)\|_F^2 \\
&- \|\widetilde{\mathbf{D}}(\tau-1) - \mathbf{D}(\tau-1)\|_F\|\mathbf{Q}(\tau-1) - \mathbf{Q}^*\|_F \\
\overset{(b)}{\geq}&\frac{1}{2\gamma}\Delta(\tau-1) - \frac{\gamma}{2}(\psi(\delta) + \sqrt{N_R}B^2)^2 - 2\psi(\delta)\bar{P}
\end{aligned}$$

where (a) follows from Fact 1 and (b) follows from Lemma 3 and the fact that $\|\mathbf{Q}(\tau-1) - \mathbf{Q}^*\|_F \leq \|\mathbf{Q}(\tau-1)\|_F + \|\mathbf{Q}^*\|_F \leq \text{tr}(\mathbf{Q}(\tau-1)) + \text{tr}(\mathbf{Q}^*) \leq 2\bar{P}$, which is implied by Fact 1, Fact 2 and fact $\mathbf{Q}(\tau-1), \mathbf{Q}^* \in \widetilde{\mathcal{Q}}$. Replacing $\tau - 1$ with τ yields for all $\tau \in \{0, 1, \ldots\}$, $R(\tau) - R^{opt}(\tau) \geq \frac{1}{2\gamma}\Delta(\tau) - \frac{\gamma}{2}(\psi(\delta) + \sqrt{N_R}B^2)^2 - 2\psi(\delta)\bar{P}$.

Fix $t > 0$. Summing over $\tau \in \{0, 1, \ldots, t-1\}$, dividing by factor t and noting that $\sum_{\tau=0}^{t-1}\Delta(\tau)$ is a telescope sum gives $\frac{1}{t}\sum_{\tau=0}^{t-1} R(\tau) - \frac{1}{t}\sum_{\tau=0}^{t-1} R^{opt}(\tau) \geq \frac{1}{2\gamma t}(\|\mathbf{Q}(t) - \mathbf{Q}^*\|_F - \|\mathbf{Q}(0) - \mathbf{Q}^*\|_F) - \frac{\gamma}{2}(\psi(\delta) + \sqrt{N_R}B^2)^2 - 2\psi(\delta)\bar{P} \geq -\frac{\bar{P}}{\gamma t} - \frac{\gamma}{2}(\psi(\delta) + \sqrt{N_R}B^2)^2 - 2\psi(\delta)\bar{P}$, where the last inequality follows because $\|\mathbf{Q}(0) - \mathbf{Q}^*\|_F \leq \|\mathbf{Q}(0)\|_F + \|\mathbf{Q}^*\|_F \leq \text{tr}(\mathbf{Q}(0)) + \text{tr}(\mathbf{Q}^*) \leq 2\bar{P}$ and $\|\mathbf{Q}(t) - \mathbf{Q}^*\|_F \geq 0$. ■

Theorem 4 proves a sample path guarantee on the average utility. It shows that the convergence time to reach an $\epsilon + 2\psi(\delta)\bar{P}$ approximate solution is $O(1/\epsilon^2)$. Note that if $\delta = 0$, then equations (14) and (15) are recovered by Theorem 4.

Theorem 4 also isolates the effects of delay and quantization. The observation is that the effect of CSIT delay vanishes as Algorithm 3 runs for a sufficiently long time. In some sense, delayed but accurate CSIT is almost as good as perfect CDIT. In contrast, the effect of CSIT quantization does not vanish as Algorithm 3 runs for a sufficiently long time. The performance degradation due to quantization scales linearly with respect to the quantization error since $\psi(\delta) \in O(\delta)$. Intuitively, this is reasonable since the power allocation based on quantized CSIT is actually optimizing another different MIMO system.

D. Extensions

1) T-Slot Delayed and Quantized CSIT: Thus far, we have assumed that CSIT is delayed by one slot. In fact, if CSIT is delayed by T slots, we can modify the update of transmit covariances in Algorithm 3 as $\mathbf{Q}(t) = \mathcal{P}_{\widetilde{\mathcal{Q}}}[\mathbf{Q}(t-T)+\gamma\widetilde{\mathbf{D}}(t-T)]$. A T-slot version of Theorem 4 can be similarly proven.

2) Algorithm 3 with Time Varying γ: Algorithm 3 can be extended to have time varying step size $\gamma(t) = \frac{1}{\sqrt{t}}$ at time t. The full version [17] proves that such an algorithm yields $\frac{1}{t}\sum_{\tau=0}^{t-1} R(\tau) \geq \frac{1}{t}\sum_{\tau=0}^{t-1} R^{opt}(\tau) - \frac{\bar{P}}{\sqrt{t}} - \frac{1}{\sqrt{t}}(\psi(\delta) + \sqrt{N_R}B^2)^2 - 2\psi(\delta)\bar{P}$ for all $t > 0$. This shows the convergence time to an $\epsilon + 2\psi(\delta)\bar{P}$ approximate solution is again $O(1/\epsilon^2)$. However, an advantage of time varying step sizes is the performance automatically gets improved as the algorithm runs and there is no need to restart the algorithm with a different constant step size if a better performance is demanded.

V. Simulations

Consider a MIMO system where both the transmitter and the receiver have two antennas. The power constraints are $\bar{P} = 5$ and $P = 10$. The channel has two realizations with equal probability 0.5, i.e., $\mathbf{H}_1 = 0.5\begin{bmatrix} e^{j0.84\pi} & e^{j1.58\pi} \\ e^{j1.83\pi} & e^{j1.97\pi} \end{bmatrix}$ and $\mathbf{H}_2 = \begin{bmatrix} e^{j1.31\pi} & e^{j1.69\pi} \\ e^{j0.07\pi} & e^{j1.86\pi} \end{bmatrix}$. If the channel is quantized, they are quantized as $\widetilde{\mathbf{H}}_1$ and $\widetilde{\mathbf{H}}_2$, respectively. The algorithms in this paper can be easily applied to examples with infinite possible outcomes for the channel matrix. This simple example of two possibilities is considered because an offline optimal solution based on perfect CDIT can only be computed when the number of samples is small. [3]

Figure 1 compares the performance of Algorithm 1 with perfect CSIT and the optimal solution to problem (1)-(3). In the simulation, we take $V = 1000$.

Figure 2 compares the performance of Algorithm 3 with one slot delayed and quantized CSIT and the optimal solution to problem (5)-(7). To study the effect of quantization error, we consider 3 different quantization levels. Case 1: $\widetilde{\mathbf{H}}_1 = 0.5\begin{bmatrix} e^{j0.8\pi} & e^{j1.5\pi} \\ e^{j1.8\pi} & e^{j1.9\pi} \end{bmatrix}$ and $\widetilde{\mathbf{H}}_2 = \begin{bmatrix} e^{j1.3\pi} & e^{j1.6\pi} \\ e^{j0\pi} & e^{j1.8\pi} \end{bmatrix}$; Case 2: $\widetilde{\mathbf{H}}_1 = 0.5\begin{bmatrix} e^{j\pi} & e^{j1.5\pi} \\ e^{j2\pi} & e^{j2\pi} \end{bmatrix}$ and $\widetilde{\mathbf{H}}_2 = \begin{bmatrix} e^{j1.5\pi} & e^{j1.5\pi} \\ e^{j0\pi} & e^{j2\pi} \end{bmatrix}$; Case 3: $\widetilde{\mathbf{H}}_1 = 0.5\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and $\widetilde{\mathbf{H}}_2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. In the simulation, we take $\mathbf{Q}(0) = \mathbf{0}$ and $\gamma = 10^{-3}$. It can be observed that performance becomes worse as CSIT quantization gets coarser, while the average power constraints are strictly satisfied even with quantized CSIT.

[3] This is known as the curse of dimensionality for stochastic optimization due to the large sample size. That is, even with perfect CDIT, problem (1)-(3) and problem (5)-(7) can be numerically hard to solve when the sample size of $\mathbf{H}$ is large. In contrast, the dynamic algorithms proposed in this paper can deal with problems even with an infinite number of samples and the performance guarantees are independent of the sample size.

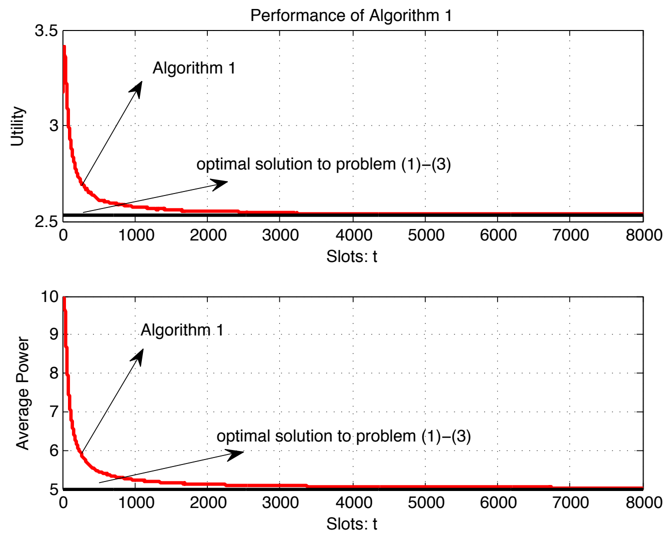

Fig. 1. Performance of Algorithm 1.

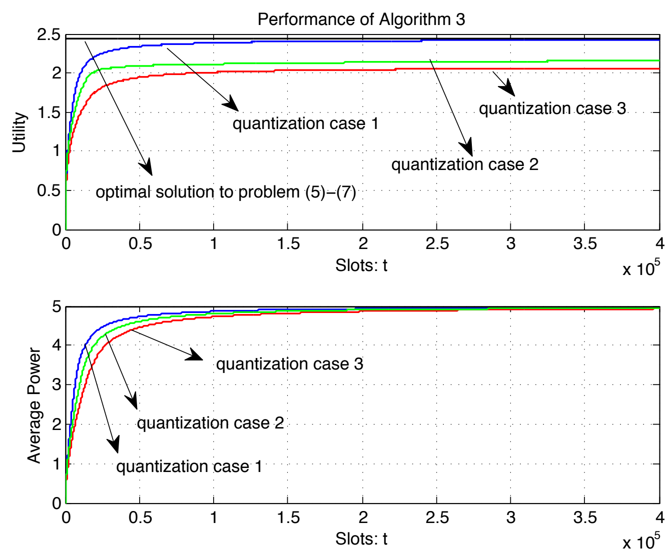

Fig. 2. Performance of Algorithm 3.

VI. Conclusion

This paper considers dynamic power allocation in MIMO fading systems without CDIT. In the case of ideal CSIT, the proposed dynamic power policy can approach optimality. In the case of delayed and quantized CSIT, the proposed dynamic power allocation policy can achieve $O(\delta)$ sub-optimality, where δ is the quantization error.

Appendix A – Proof of Theorem 1

The proof method is an extension of Section 3.2 in [1], which gives the structure of the optimal transmit covariance in deterministic MIMO channels.

Note that $\log\det(\mathbf{I}+\mathbf{H}\mathbf{Q}\mathbf{H}^H) \stackrel{(a)}{=} \log\det(\mathbf{I}+\mathbf{Q}\mathbf{H}^H\mathbf{H}) \stackrel{(b)}{=} \log\det(\mathbf{I}+\mathbf{Q}\mathbf{U}^H\mathbf{\Sigma}\mathbf{U}) \stackrel{(c)}{=} \log\det(\mathbf{I}+\mathbf{\Sigma}^{1/2}\mathbf{U}\mathbf{Q}\mathbf{U}^H\mathbf{\Sigma}^{1/2})$, where (a) and (c) follows from the elementary identity

$\log\det(\mathbf{I}_n + \mathbf{AB}) = \log\det(\mathbf{I}_n + \mathbf{BA}), \forall \mathbf{A}, \mathbf{B} \in \mathbb{C}^{n\times n}$; and (b) follows from the fact that $\mathbf{H}^H\mathbf{H} = \mathbf{U}^H\mathbf{\Sigma}\mathbf{U}$. Define $\widetilde{\mathbf{Q}} = \mathbf{U}\mathbf{Q}\mathbf{U}^H$, which is semidefinite positive if and only if $\mathbf{Q}$ is. Note that $\text{tr}(\widetilde{\mathbf{Q}}) = \text{tr}(\mathbf{U}\mathbf{Q}\mathbf{U}^H) = \text{tr}(\mathbf{Q})$ by the fact that $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA}), \forall \mathbf{A} \in \mathbb{C}^{m\times n}, \mathbf{B} \in \mathbb{C}^{n\times m}$. Thus, problem (11)-(13) is equivalent to

$$\max_{\widetilde{\mathbf{Q}}} \quad \log\det(\mathbf{I} + \mathbf{\Sigma}^{1/2}\widetilde{\mathbf{Q}}\mathbf{\Sigma}^{1/2}) - \frac{Z}{V}\text{tr}(\widetilde{\mathbf{Q}}) \tag{21}$$

$$\text{s.t.} \quad \text{tr}(\widetilde{\mathbf{Q}}) \leq P \tag{22}$$

$$\widetilde{\mathbf{Q}} \in \mathbb{S}_+^{N_T} \tag{23}$$

Fact 5 **(**Hadamard's Inequality, Theorem 7.8.1 in [22]**).** *For all $\mathbf{A} \in \mathbb{S}_+^n$, $\det(\mathbf{A}) \leq \prod_{i=1}^n A_{ii}$ with equality if $\mathbf{A}$ is diagonal.*

The next claim can be proven using Hadamard's inequality.

Claim 1. *Problem* (21)-(23) *has a diagonal optimal solution.*

Proof: Suppose problem (21)-(23) has a non-diagonal optimal solution given by matrix $\widetilde{\mathbf{Q}}$. Consider a diagonal matrix $\widehat{\mathbf{Q}}$ whose entries are identical to the diagonal entries of $\widetilde{\mathbf{Q}}$. Note that $\text{tr}(\widehat{\mathbf{Q}}) = \text{tr}(\widetilde{\mathbf{Q}})$. To show $\widehat{\mathbf{Q}}$ is a solution no worse than $\widetilde{\mathbf{Q}}$, it suffices to show that $\log\det(\mathbf{I} + \mathbf{\Sigma}^{1/2}\widehat{\mathbf{Q}}\mathbf{\Sigma}^{1/2}) \geq \log\det(\mathbf{I} + \mathbf{\Sigma}^{1/2}\widetilde{\mathbf{Q}}\mathbf{\Sigma}^{1/2})$. This is true becase $\det(\mathbf{I} + \mathbf{\Sigma}^{1/2}\widehat{\mathbf{Q}}\mathbf{\Sigma}^{1/2}) = \prod_{i=1}^{N_T}(1 + \widehat{Q}_{ii}\sigma_i) = \prod_{i=1}^{N_T}(1 + \widetilde{Q}_{ii}\sigma_i) \geq \det(\mathbf{I} + \mathbf{\Sigma}^{1/2}\widetilde{\mathbf{Q}}\mathbf{\Sigma}^{1/2})$, where the last inequality follows from Hadamard's inequality. Thus, $\widehat{\mathbf{Q}}$ is a solution no worse than $\widetilde{\mathbf{Q}}$ and hence optimal. ∎

By Claim 1, we can consider $\widetilde{\mathbf{Q}} = \Theta = \text{diag}(\theta_1, \theta_2, \ldots, \theta_{N_T})$ and problem (21)-(23) is equivalent to

$$\max \quad \sum_{i=1}^{N_T}\log(1 + \theta_i\sigma_i) - \frac{Z}{V}\sum_{i=1}^{N_T}\theta_i \tag{24}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_T}\theta_i \leq P \tag{25}$$

$$\theta_i \geq 0, \forall i \in \{1, 2, \ldots, N_T\} \tag{26}$$

Note that problem (24)-(26) satisfies Slater's condition. So the optimal solution to problem (24)-(26) is characterized by KKT conditions [20]. The remaining part is similar to the derivation of the water-filling solution of power allocation in parallel channels, e.g., the proof of Example 5.2 in [20]. Introducing Lagrange multipliers $\mu \in \mathbb{R}_+$ for inequality constraint $\sum_{i=1}^{N_T}\theta_i \leq P$ and $\boldsymbol{\nu} = [\nu_1, \ldots, \nu_{N_T}]^T \in \mathbb{R}_+^{N_T}$ for inequality constraints $\theta_i \geq 0, i \in \{1, 2, \ldots, N_T\}$. Let $\boldsymbol{\theta}^* = [\theta_1^*, \ldots, \theta_{N_T}^*]^T$ and $(\mu^*, \boldsymbol{\nu}^*)$ be any primal and dual optimal points with zero duality gap. By KKT conditions, we have $-\frac{\sigma_i}{1+\theta_i^*\sigma_i} + Z/V + \mu^* - \nu_i^* = 0, \forall i \in \{1, 2, \ldots, N_T\}; \sum_{i=1}^{N_T}\theta_i^* \leq P; \mu^* \geq 0; \mu^*\big[\sum_{i=1}^{N_T}\theta_i^* - P\big] = 0; \theta_i^* \geq 0, \forall i \in \{1, 2, \ldots, N_T\}; \nu_i^* \geq 0, \forall i \in \{1, 2, \ldots, N_T\}; \nu_i^*\theta_i^* = 0, \forall i \in \{1, 2, \ldots, N_T\}$.

Eliminating $\nu_i^*, \forall i \in \{1, 2, \ldots, N_T\}$ in all equations yields $\mu^* + Z/V \geq \frac{\sigma_i}{1+\theta_i^*\sigma_i}, \forall i \in \{1, 2, \ldots, N_T\}; \sum_{i=1}^{N_T}\theta_i^* \leq P; \mu^* \geq 0; \mu^*\big[\sum_{i=1}^{N_T}\theta_i^* - P\big] = 0; \theta_i^* \geq 0, \forall i \in \{1, 2, \ldots, N_T\}; (\mu^* + Z/V - \frac{\sigma_i}{1+\theta_i^*\sigma_i})\theta_i^* = 0, \forall i \in \{1, 2, \ldots, N_T\}$.

For all $i \in \{1, 2, \ldots, N_T\}$, we consider $\mu^* + Z/V < \sigma_i$ and $\mu^* + Z/V \geq \sigma_i$ separately:

1) If $\mu^* + Z/V < \sigma_i$, then $\mu^* + Z/V \geq \frac{\sigma_i}{1+\theta_i^*\sigma_i}$ holds only when $\theta_i^* > 0$, which by $(\mu^* + Z/V - \frac{\sigma_i}{1+\theta_i^*\sigma_i})\theta_i^*$ implies that $\mu^* + Z/V - \frac{\sigma_i}{1+\theta_i^*\sigma_i} = 0$, i.e., $\theta_i^* = \frac{1}{\mu^*+Z/V} - \frac{1}{\sigma_i}$.
2) If $\mu^* + Z/V \geq \sigma_i$, then $\theta_i^* > 0$ is impossible, because $\theta_i^* > 0$ implies that $\mu^* + Z/V - \frac{\sigma_i}{1+\theta_i^*\sigma_i} > 0$, which together with $\theta_i^* > 0$ contradict the slackness condition $(\mu^* + Z/V - \frac{\sigma_i}{1+\theta_i^*\sigma_i})\theta_i^* = 0$. Thus, if $\mu^* + Z/V \geq \sigma_i$, we must have $\theta_i^* = 0$.

Summarizing both cases, we have $\theta_i^* = \max\big[0, \frac{1}{\mu^*+Z/V} - \frac{1}{\sigma_i}\big], \forall i \in \{1, 2, \ldots, N_T\}$, where μ^* is chosen such that $\sum_{i=1}^n\theta_i^* \leq P$, $\mu^* \geq 0$ and $\mu^*\big[\sum_{i=1}^{N_T}\theta_i^* - P\big] = 0$.

To find such μ^*, we first check if $\mu^* = 0$. If $\mu^* = 0$ is true, the slackness condition $\mu^*\big[\sum_{i=1}^{N_T}\theta_i^* - P\big] = 0$ is guaranteed to hold and we need to further require $\sum_{i=1}^{N_T}\theta_i^* = \sum_{i=1}^{N_T}\max\big[0, \frac{1}{\mu^*+Z/V} - \frac{1}{\sigma_i}\big] \leq P$. Thus $\mu^* = 0$ if and only if $\sum_{i=1}^{N_T}\max\big[0, \frac{1}{Z/V} - \frac{1}{\sigma_i}\big] \leq P$. Thus, Algorithm 2 checks if $\sum_{i=1}^{N_T}\max\big[0, \frac{1}{Z/V} - \frac{1}{\sigma_i}\big] \leq P$ holds at the first step and if this is true, then we conclude $\mu^* = 0$ and we are done!

Otherwise, we know $\mu^* > 0$. By the slackness condition $\mu^*\big[\sum_{i=1}^{N_T}\theta_i^* - P\big] = 0$, we must have $\sum_{i=1}^{N_T}\theta_i^* = \sum_{i=1}^{N_T}\max\big[0, \frac{1}{\mu^*+Z/V} - \frac{1}{\sigma_i}\big] = P$. To find $\mu^* > 0$ such that $\sum_{i=1}^{N_T}\max\big[0, \frac{1}{\mu^*+Z/V} - \frac{1}{\sigma_i}\big] = P$, we could apply a bisection search by noting that all θ_i^* are decreasing with respect to μ^*.

Another algorithm of finding μ^* is inspired by the observation that if $\sigma_j \geq \sigma_k, \forall j, k \in \{1, 2, \ldots, N_T\}$, then $\theta_j^* \geq \theta_k^*$. Thus, we first sort all σ_i in a decreasing order, say π is the permutation such that $\sigma_{\pi(1)} \geq \sigma_{\pi(2)} \geq \cdots \geq \sigma_{\pi(N_T)}$; and then sequentially check if $i \in \{1, 2, \ldots, N_T\}$ is the index such that $\sigma_{\pi(i)} - \mu^* \geq 0$ and $\sigma_{\pi(i+1)} - \mu^* \leq 0$. To check this, we first assume i is indeed such an index and solve the equation $\sum_{j=1}^i\big[\frac{1}{\mu^*+Z/V} - \frac{1}{\sigma_{\pi(j)}}\big] = P$ to obtain μ^*; (Note that in Algorithm 2, to avoid recalculating the partial sum $\sum_{j=1}^i\frac{1}{\sigma_{\pi(j)}}$ for each i, we introduce the parameter $S_i = \sum_{j=1}^i\frac{1}{\sigma_{\pi(j)}}$ and update S_i incrementally. By doing this, the complexity of each iteration in the loop is only $O(1)$.) then verify the assumption by checking if $\frac{1}{\mu^*+Z/V} - \frac{1}{\sigma_{\pi(i)}} \geq 0$ and $\frac{1}{\mu^*+Z/V} - \frac{1}{\sigma_{\pi(i+1)}} \leq 0$. This algorithm is described in Algorithm 2.

Appendix B – Proof of Theorem 3

Claim 2. *If $\widehat{\mathbf{\Theta}}$ is an optimal solution to the following convex program:*

$$\min \quad \frac{1}{2}\|\mathbf{\Theta} - \mathbf{\Sigma}\|_F^2 \tag{27}$$

$$\textit{s.t.} \quad tr(\mathbf{\Theta}) \leq \bar{P} \tag{28}$$

$$\mathbf{\Theta} \in \mathbb{S}_+^{N_T} \tag{29}$$

then $\widehat{\mathbf{Q}} = \mathbf{U}^H\widehat{\mathbf{\Theta}}\mathbf{U}$ is an optimal solution to problem (16)-(18)*.*

Proof: This claim can be proven by contradiction. Let $\widehat{\mathbf{\Theta}}$ be an optimal solution to convex program (27)-(29) and define $\widehat{\mathbf{Q}} = \mathbf{U}^H\widehat{\mathbf{\Theta}}\mathbf{U}$. Assume that there exists $\widetilde{\mathbf{Q}} \in \mathbb{S}_+^{N_T}$ such that $\widetilde{\mathbf{Q}} \neq \widehat{\mathbf{Q}}$ and is a solution to problem (16)-(18) that is strictly better than $\widehat{\mathbf{Q}}$. Consider $\widetilde{\mathbf{\Theta}} = \mathbf{U}\widetilde{\mathbf{Q}}\mathbf{U}^H$ and reach a contradiction by showing $\widetilde{\mathbf{\Theta}}$ is strictly better than $\widehat{\mathbf{\Theta}}$ as follows:

Note that $\text{tr}(\widetilde{\mathbf{\Theta}}) = \text{tr}(\mathbf{U}\widetilde{\mathbf{Q}}\mathbf{U}^H) = \text{tr}(\widetilde{\mathbf{Q}}) \leq \bar{P}$, where the last inequality follows from the assumption that $\widetilde{\mathbf{Q}}$ is solution to problem (16)-(18). Also note that $\widetilde{\mathbf{\Theta}} \in \mathbb{S}_+^{N_T}$ since $\widetilde{\mathbf{Q}} \in \mathbb{S}_+^{N_T}$. Thus, $\widetilde{\mathbf{\Theta}}$ is feasible to problem (27)-(29).

Note that $\|\widetilde{\mathbf{\Theta}} - \mathbf{\Sigma}\|_F \overset{(a)}{=} \|\mathbf{U}^H\widetilde{\mathbf{\Theta}}\mathbf{U} - \mathbf{U}^H\mathbf{\Sigma}\mathbf{U}\|_F \overset{(b)}{=} \|\widetilde{\mathbf{Q}} - \mathbf{X}\|_F \overset{(c)}{<} \|\widehat{\mathbf{Q}} - \mathbf{X}\|_F \overset{(d)}{=} \|\mathbf{U}\widehat{\mathbf{Q}}\mathbf{U}^H - \mathbf{U}\mathbf{X}\mathbf{U}^H\|_F \overset{(e)}{=} \|\widehat{\mathbf{\Theta}} - \mathbf{\Sigma}\|_F$, where (a) and (d) follow from the fact Frobenius norm is unitary invariant[4]; (b) follows from the fact that $\widetilde{\mathbf{\Theta}} = \mathbf{U}\widetilde{\mathbf{Q}}\mathbf{U}^H$ and $\mathbf{X} = \mathbf{U}^H\mathbf{\Sigma}\mathbf{U}$; (c) follows from the fact that $\widetilde{\mathbf{Q}}$ is strictly better than $\widehat{\mathbf{Q}}$; and (e) follows from the fact that $\widehat{\mathbf{Q}} = \mathbf{U}^H\widehat{\mathbf{\Theta}}\mathbf{U}$ and $\mathbf{X} = \mathbf{U}^H\mathbf{\Sigma}\mathbf{U}$. Thus, $\widetilde{\mathbf{\Theta}}$ is strictly better than $\widehat{\mathbf{\Theta}}$. A contradiction! ∎

Claim 3. *The optimal solution to problem* (27)-(29) *must be a diagonal matrix.*

Proof: This claim can be proven by contradiction. Assume that problem (27)-(29) has an optimal solution $\widetilde{\mathbf{\Theta}}$ that is not diagonal. Since $\widetilde{\mathbf{\Theta}}$ is positive semidefinite, all the diagonal entries of $\widetilde{\mathbf{\Theta}}$ are non-negative. Define $\widehat{\mathbf{\Theta}}$ as a diagonal matrix whose the i-th diagonal entry is equal to the i-th diagonal entry of $\widetilde{\mathbf{\Theta}}$ for all $i \in \{1, 2, \ldots, N_T\}$. Note that $\text{tr}(\widehat{\mathbf{\Theta}}) = \text{tr}(\widetilde{\mathbf{\Theta}}) \leq P$ and $\widehat{\mathbf{\Theta}} \in \mathbb{S}_+^n$. Thus, $\widehat{\mathbf{\Theta}}$ is feasible to problem (27)-(29). Note that $\|\widehat{\mathbf{\Theta}} - \mathbf{\Sigma}\|_F < \|\widetilde{\mathbf{\Theta}} - \mathbf{\Sigma}\|_F$ since $\mathbf{\Sigma}$ is diagonal. Thus, $\widehat{\mathbf{\Theta}}$ is a solution strictly better than $\widetilde{\mathbf{\Theta}}$. A contradiction! So the optimal solution to problem (27)-(29) must be a diagonal matrix. ∎

By the above two claims, it suffices to assume that the optimal solution to problem (16)-(18) has the structure $\hat{\mathbf{Q}} = \mathbf{U}^H\mathbf{\Theta}\mathbf{U}$, where Θ is a matrix with non-negative entries. To solve problem (16)-(18), it suffices to consider the following convex program.

$$\min \quad \frac{1}{2}\sum_{i=1}^{N_T}(\theta_i - \sigma_i)^2 \tag{30}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_T}\theta_i \leq \bar{P} \tag{31}$$

$$\theta_i \geq 0, \forall i \in \{1, 2, \ldots, N_T\} \tag{32}$$

Note that problem (30)-(32) satisfies Slater's condition. So the optimal solution to problem (30)-(32) is characterized by KKT conditions [20]. The remaining part is similar to the proof of Theorem 1 and can be found in the full version [17].

References

[1] I. E. Telatar, "Capacity of multi-antenna gaussian channels," *European transactions on telecommunications*, vol. 10, no. 6, pp. 585–596, 1999.

[4] That is $\|\mathbf{AU}\|_F = \|\mathbf{A}\|_F$ for all $\mathbf{A} \in \mathbb{C}^{n\times n}$ and all unitary matrix $\mathbf{U}$.

[2] S. K. Jayaweera and H. V. Poor, "Capacity of multiple-antenna systems with both receiver and transmitter channel state information," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2697–2709, 2003.

[3] S. A. Jafar, S. Vishwanath, and A. Goldsmith, "Channel capacity and beamforming for multiple transmit and receive antennas with covariance feedback," in *Proceedings of IEEE International Conference on Communications (ICC)*, 2001.

[4] E. Jorswieck and H. Boche, "Channel capacity and capacity-range of beamforming in MIMO wireless systems under correlated fading with covariance feedback," *IEEE Transactions on Wireless Communications*, vol. 3, no. 5, pp. 1543–1553, 2004.

[5] V. V. Veeravalli, Y. Liang, and A. M. Sayeed, "Correlated MIMO wireless channels: capacity, optimal signaling, and asymptotics," *IEEE Transactions on Information Theory*, vol. 51, no. 6, pp. 2058–2072, 2005.

[6] M. J. Neely, "Dynamic power allocation and routing for satellite and wireless networks with time varying channels," Ph.D. dissertation, Massachusetts Institute of Technology, 2003.

[7] ——, *Stochastic network optimization with application to communication and queueing systems*. Morgan & Claypool Publishers, 2010, vol. 3, no. 1.

[8] J.-W. Lee, R. R. Mazumdar, and N. B. Shroff, "Opportunistic power scheduling for dynamic multi-server wireless systems," *IEEE Transactions on Wireless Communications*, vol. 5, no. 6, pp. 1506–1515, 2006.

[9] A. Ribeiro, "Ergodic stochastic optimization algorithms for wireless communication and networking," *IEEE Transactions on Signal Processing*, vol. 58, no. 12, pp. 6369–6386, 2010.

[10] Y. Hu and A. Ribeiro, "Adaptive distributed algorithms for optimal random access channels," *IEEE Transactions on Wireless Communications*, vol. 10, no. 8, pp. 2703–2715, 2011.

[11] J. Liu, Y. T. Hou, Y. Shi, and H. D. Sherali, "On performance optimization for multi-carrier mimo ad hoc networks," in *Proceedings of the 10th ACM international symposium on Mobile ad hoc networking and computing (MobiHoc)*, 2009, pp. 43–54.

[12] M. Zinkevich, "Online convex programming and generalized infinitesimal gradient ascent," in *Proceedings of the 20th International Conference on Machine Learning (ICML)*, 2003.

[13] N. Buchbinder, L. Lewin-Eytan, I. Menache, J. S. Naor, and A. Orda, "Dynamic power allocation under arbitrary varying channels-an online approach," in *Proceedings of IEEE Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, 2009.

[14] ——, "Dynamic power allocation under arbitrary varying channels – the multi-user case," in *Proceedings of IEEE Annual Joint Conference of the IEEE Computer and Communications Societies (INFOCOM)*, 2010.

[15] I. Stiakogiannakis, P. Mertikopoulos, and C. Touati, "Adaptive power allocation and control in time-varying multi-carrier MIMO networks," *arXiv preprint arXiv:1503.02155*, 2015.

[16] A. Goldsmith, *Wireless communications*. Cambridge University Press, 2005.

[17] H. Yu and M. J. Neely, "Dynamic power allocation in MIMO fading systems without channel distribution information," *arXiv preprint arXiv:1512.08419*, 2015.

[18] A. Feiten, S. Hanly, and R. Mathar, "Derivatives of mutual information in gaussian vector channels with applications," in *Proceedings of IEEE International Symposium on Information Theory (ISIT)*, 2007, pp. 2296–2300.

[19] D. P. Palomar and M. A. Lagunas, "Joint transmit-receive space-time equalization in spatially correlated MIMO channels: A beamforming approach," *IEEE Journal on Selected Areas in Communications*, vol. 21, no. 5, pp. 730–743, 2003.

[20] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[21] S. Boyd and L. Xiao, "Least-squares covariance matrix adjustment," *SIAM Journal on Matrix Analysis and Applications*, vol. 27, no. 2, pp. 532–546, 2005.

[22] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.